

Branching Fractions and Direct CP Asymmetries of

$$\overline{B}_s^0 \rightarrow K^0 h^+ h'^- (h^{(\prime)} = K, \pi) \text{ Decays}$$

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Motivated by the recent LHCb collaboration measurements of charmless three-body decays of \overline{B}_s^0 meson, we calculate the branching fractions of $\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$, $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$, $\overline{B}_s^0 \rightarrow K^0 \pi^+ K^-$ and $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$ decay modes using the factorization approach. Both the resonant and nonresonant contributions are studied in detail. For the decays $\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ and $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$, our results agree well with experimental data, and the former is dominated by the K^* , while the latter one is dominated by the nonresonant contribution. Considering the flavor $SU(3)$ symmetry violation, the sum of branching fractions of $\overline{B}_s^0 \rightarrow K^0 \pi^+ K^-$ and $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$ could accommodate the data well too. It should be noted that both branching fractions are sensitive to the scalar density $\langle K\pi | \bar{s}q | 0 \rangle$. Furthermore, the resonant contributions are dominated by the scalar $K_0^*(1430)$. We hope that these branching fractions could be measured individually in the experiments so as to test the factorization approach and the flavor $SU(3)$ asymmetry. Moreover, the direct CP asymmetries of these decays are also investigated, which could be measured in the running LHCb experiment and Super-b factory in the future.

Key Words: B_s^0 three-body decay, Factorization Approach, CP violation

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1 Introduction

In the recent years, the charmless three-body decays of B mesons have attached a great deal of attention, because by studying them one can determine the Cabibbo-Kabayashi-Maskawa (CKM) parameters or search for the possible new physics effect beyond the standard model. For example, the Dalitz-plot analysis combined with flavor SU(3) symmetry allows us to extract the angle γ cleanly from $B \rightarrow K\pi\pi$ and $B \rightarrow KKK$ decays [1, 2, 3]. However, the three-body decays of B mesons are more complicated than the two-body cases, because both resonant (vector or scalar) and nonresonant contributions involve the hadronic matrix elements. The interference between resonant and nonresonant amplitudes makes it rather hard to disentangle these distinct contributions and extract the nonresonant one, so it is very difficult to measure the direct three-body decays experimentally. Over the recent years, thanks to the two B factories and LHCb experiment, remarkable progress in measuring the branching fractions and direct CP asymmetries of the three-body decays has been made by using the Dalitz-plot analysis (for a review see ref. [4]).

On the theoretical side, the charmless three-body decays of heavy mesons have been studied within the different approaches, such as the factorization approach (FA) [5, 6, 7, 8, 9, 10, 11, 12, 13], diagrammatic approach combined with SU(3) symmetry [14, 15, 16, 17], perturbative QCD approach [18, 19], and other approaches [20, 21, 22]. FA, based on the phenomenological factorization model, has been applied in calculating three-body decays of heavy meson widely, although factorization has not been proved in the three-body decays. Within the FA, most predicted branching fractions and direct CP asymmetries of $B \rightarrow PPP$ decays [9, 10, 11, 12, 13] agree with the experimental data well, except for decay $\overline{B}^0 \rightarrow K^+K^-\pi^0$.

Here, we will review the FA briefly by taking $B^- \rightarrow \pi^+\pi^-\pi^-$ as an example. Under the FA, the amplitude of decay $B^- \rightarrow \pi^+\pi^-\pi^-$ is usually split into three distinct factorizable terms: (i) the current-induced process with a meson emission, $\langle B^- \rightarrow \pi^+\pi^- \rangle \times \langle 0 \rightarrow \pi^- \rangle$, (ii) the transition process, $\langle B^- \rightarrow \pi^- \rangle \times \langle 0 \rightarrow \pi^+\pi^- \rangle$, and (iii) the annihilation process $\langle B^- \rightarrow 0 \rangle \times \langle 0 \rightarrow \pi^+\pi^-\pi^- \rangle$, where $\langle A \rightarrow B \rangle$ stands for an $A \rightarrow B$ transition matrix element. One of the nonresonant contributions due to $\langle B^- \rightarrow \pi^+\pi^- \rangle$ has been studied on the basis of the heavy meson chiral perturbative theory (HMChPT) [23, 24, 25, 26], although applicability of this framework in the whole kinematics region is still controversial [27]. However, it could lead to large branching fraction ($\mathcal{O}(10^{-5})$) [5, 6], which disagrees with the experimental data (5.3×10^{-6}) from BaBar [28]. In fact, this issue can be understood considering the applicability of the HMChPT. When the HMChPT is applied to three-body decays, two of the final-state pseudoscalars should be soft. If the soft meson result is assumed to be the same in the whole Dalitz plot, the decay rate will be greatly overestimated. To overcome this issue, Cheng *et al.* proposed in refs [10, 11, 12, 13] to parameterize the momentum dependence of nonresonant amplitudes $\langle B \rightarrow PP \rangle$ in an exponential form $e^{-\alpha_{\text{NR}} p_B \cdot (p_i + p_j)}$ so that the HMChPT results are recovered in the soft pseudoscalar meson limit. The tree-dominated $B^- \rightarrow \pi^+\pi^-\pi^-$ decay data is used to fix the unknown parameter α_{NR} . Besides from the current-induced process, the matrix elements $\langle \pi^+\pi^- | \bar{q}\gamma_\mu q | 0 \rangle$ and $\langle \pi^+\pi^- | \bar{d}d | 0 \rangle$ also receive nonresonant contributions. In principle, the weak vector form factor of the former matrix element can be related to the charged pion electromagnetic (e.m.) form factors. However, unlike the kaon case, the time-like e.m. form factors of the pions are not measured well

enough allowing us to determine the nonresonant parts. Therefore, the nonresonant contribution to $\langle \pi^+ \pi^- | \bar{q} \gamma_\mu q | 0 \rangle$ is always ignored. The matrix element $\langle \pi^+ \pi^- | \bar{d} d | 0 \rangle$ is related to $\langle K^+ K^- | \bar{s} s | 0 \rangle$ via SU(3) flavor symmetry. As for the resonant contributions to three-body decays, vector and scalar resonances contribute to the two-body matrix elements $\langle P_1 P_2 | V_\mu | 0 \rangle$ and $\langle P_1 P_2 | S | 0 \rangle$, respectively. They can also contribute to the three-body matrix element $\langle P_1 P_2 | V_\mu - A_\mu | 0 \rangle$. Resonant effects are described in terms of the usual Breit-Wigner formalism. In this manner, the relevant resonances which contribute to the 3-body decays of interest could be figured out. In conjunction with the nonresonant contribution, the total rates for three-body decays are well calculated.

Very recently, corresponding to an integrated luminosity of 1.0 fb^{-1} recorded at a centre-of-mass energy of 7 TeV, LHCb collaboration published their first measurements of the branching fractions of three-body decays of B_s^0 meson [29] as follows:

$$Br(B_s^0 \rightarrow K^0 \pi^+ \pi^-) = (14.3 \pm 2.8 \pm 1.8 \pm 0.6) \times 10^{-6}, \quad (1)$$

$$Br(B_s^0 \rightarrow K^0 K^\pm \pi^\mp) = (73.6 \pm 5.7 \pm 6.9 \pm 3.0) \times 10^{-6}, \quad (2)$$

$$Br(B_s^0 \rightarrow K^0 K^+ K^-) \in [0.2; 3.4] \times 10^{-6} \text{ at } 90\% \text{ CL}. \quad (3)$$

Since these decays have never been explored before, we will calculate the branching fractions in this work using the FA proposed by Cheng *et.al.* so as to test FA in \bar{B}_s^0 decays. The resonant and nonresonant contributions of these decays will be studied, which are important in measuring the branching fractions of $\bar{B}_s^0 \rightarrow KV$ and $\bar{B}_s^0 \rightarrow KS$ experimentally. Furthermore, we will calculate the CP asymmetries of these decays, which may be helpful to extract the CKM angle γ . All results could be checked in the current LHCb experiment and Super-b factory in the future.

In the following work, we will systematically use the FA to calculate the $\bar{B}_s^0 \rightarrow K^0 h^+ h'^-$ and present the formulas in Sec. 2. The numerical results and some discussions are given in Sec. 3. We will summarize this work in Sec. 4 lastly.

2 Analytic Formalism

2.1 The Effective Hamiltonian

Under the factorization hypothesis, the matrix elements of the decay amplitudes are given by

$$\langle P_1 P_2 P_3 | \mathcal{H}_{\text{eff}} | \bar{B}_s^0 \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(r)} \langle P_1 P_2 P_3 | T_p^{(r)} | \bar{B}_s^0 \rangle, \quad (4)$$

where $\lambda_p^{(r)} \equiv V_{pb}V_{pr}^*$ with $r = d, s$. For $K\pi\pi$ and KKK modes, $r = d$; and for $KK\pi$ channels, $r = s$. The Hamiltonian $T_p^{(r)}$ has the expression [30]

$$\begin{aligned}
T_p^{(r)} = & a_1\delta_{pu}(\bar{u}b)_{V-A} \otimes (\bar{r}u)_{V-A} + a_2\delta_{pu}(\bar{r}b)_{V-A} \otimes (\bar{u}u)_{V-A} + a_3(\bar{r}b)_{V-A} \otimes \sum_q(\bar{q}q)_{V-A} \\
& + a_4^p \sum_q(\bar{q}b)_{V-A} \otimes (\bar{r}q)_{V-A} + a_5(\bar{r}b)_{V-A} \otimes \sum_q(\bar{q}q)_{V+A} \\
& - 2a_6^p \sum_q(\bar{q}b)_{S-P} \otimes (\bar{r}q)_{S+P} + a_7(\bar{r}b)_{V-A} \otimes \sum_q \frac{3}{2}e_q(\bar{q}q)_{V+A} \\
& - 2a_8^p \sum_q(\bar{q}b)_{S-P} \otimes \frac{3}{2}e_q(\bar{r}q)_{S+P} + a_9(\bar{r}b)_{V-A} \otimes \sum_q \frac{3}{2}e_q(\bar{q}q)_{V-A} \\
& + a_{10}^p \sum_q(\bar{q}b)_{V-A} \otimes \frac{3}{2}e_q(\bar{r}q)_{V-A},
\end{aligned} \tag{5}$$

with $(\bar{q}q')_{V\pm A} \equiv \bar{q}\gamma_\mu(1 \pm \gamma_5)q'$, $(\bar{q}q')_{S\pm P} \equiv \bar{q}(1 \pm \gamma_5)q'$ and a summation over $q = u, d, s$ being implied. For the effective Wilson coefficients at the renormalization scale $\mu = 2.1$ GeV, we shall follow [12] and use

$$\begin{aligned}
a_1 &\approx 0.99 + 0.037i, & a_2 &\approx 0.19 - 0.11i, & a_3 &\approx -0.002 + 0.004i, & a_5 &\approx 0.0054 - 0.005i, \\
a_4^u &\approx -0.03 - 0.02i, & a_4^c &\approx -0.04 - 0.008i, & a_6^u &\approx -0.06 - 0.02i, & a_6^c &\approx -0.06 - 0.006i, \\
a_7 &\approx 0.54 \times 10^{-4}i, & a_8^u &\approx (4.5 - 0.5i) \times 10^{-4}, & a_8^c &\approx (4.4 - 0.3i) \times 10^{-4}, \\
a_9 &\approx -0.010 - 0.0002i, & a_{10}^u &\approx (-58.3 + 86.1i) \times 10^{-5}, & a_{10}^c &\approx (-60.3 + 88.8i) \times 10^{-5},
\end{aligned} \tag{6}$$

In the above coefficients, the strong phases are from vertex corrections and penguin contractions, which have been calculated within the QCD factorization approach [31].

2.2 $\bar{B}_s^0 \rightarrow K^0\pi^+\pi^-$

With the effective Hamiltonian and the equation of motion, we obtain the $\bar{B}_s^0 \rightarrow K^0\pi^+\pi^-$ decay amplitude as

$$\begin{aligned}
\langle K^0\pi^+\pi^- | T_p | \bar{B}_s^0 \rangle = & \langle K^0\pi^+ | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle [a_1\delta_{pu} + a_4^p + a_{10}^p - r_\chi^\pi(a_6^p + a_8^p)] \\
& + \langle \pi^+\pi^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^- | (\bar{d}s)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2}a_{10}^p - r_\chi^K(a_6^p - \frac{1}{2}a_8^p) \right] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+\pi^- | (\bar{u}u)_{V-A} | 0 \rangle [a_2\delta_{pu} + a_3 + a_5 + a_7 + a_9] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+\pi^- | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 + a_4^p + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^p) \right] \\
& + \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle \pi^+\pi^- | (\bar{s}s)_{V-A} | 0 \rangle \left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9) \right] \\
& + \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle \pi^+\pi^- | \bar{d}d | 0 \rangle [-2a_6^p + a_8^p] \\
& + \langle K^0\pi^+\pi^- | (\bar{s}d)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2}a_{10}^p \right] \\
& + \langle K^0\pi^+\pi^- | \bar{s}(1 + \gamma_5)d | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle [2a_6^p - a_8^p],
\end{aligned} \tag{7}$$

where $r_\chi^\pi(\mu) = \frac{2m_\pi^2}{m_b(\mu)(m_d(\mu) - m_u(\mu))}$. It should be noted that $\langle \pi^+\pi^- | (\bar{d}d)_{V-A} | 0 \rangle = -\langle \pi^+\pi^- | (\bar{u}u)_{V-A} | 0 \rangle$ because of isospin symmetry. Besides, the matrix element $\langle \pi^+\pi^- | (\bar{s}s)_{V-A} | 0 \rangle$ is suppressed heavily by the Okubo-Zweig-Iizuka

(OZI) rule. Moreover, there exist two weak annihilation contributions, where the \overline{B}_s^0 meson is annihilated into vacuum and a final state with three mesons is then created, as the last two term are shown in the above equation. However, from the results of $B \rightarrow PPP$ decays, the contributions from annihilations are fairly small because of power and α_s suppressions, so we will ignore them in the numerical calculations in the current work.¹

For the current-induced process, the three-body matrix element $\langle K^0 \pi^+ | (\bar{u}b)_{V-A} | \overline{B}_s^0 \rangle$ could be parameterized as [26]

$$\begin{aligned} \langle K^0(p_1) \pi^+(p_2) | (\bar{u}b)_{V-A} | \overline{B}_s^0(p_B) \rangle &= ir(p_B - p_1 - p_2)_\mu + i\omega_+(p_2 + p_1)_\mu + i\omega_-(p_2 - p_1)_\mu \\ &\quad + h \epsilon_{\mu\nu\alpha\beta} p_B^\nu (p_2 + p_1)^\alpha (p_2 - p_1)^\beta. \end{aligned} \quad (8)$$

The form factors ω_\pm and r have the expressions as [26]

$$\begin{aligned} \omega_+ &= -\frac{g}{f_\pi f_K} \frac{f_{B^*} m_{B^*} \sqrt{m_{B_s} m_{B^*}}}{s_{23} - m_{B^*}^2} \left[1 - \frac{(p_B - p_1) \cdot p_1}{m_{B^*}^2} \right] + \frac{f_{B_s}}{2f_\pi f_K}, \\ \omega_- &= \frac{g}{f_\pi f_K} \frac{f_{B^*} m_{B^*} \sqrt{m_{B_s} m_{B^*}}}{s_{23} - m_{B^*}^2} \left[1 + \frac{(p_B - p_1) \cdot p_1}{m_{B^*}^2} \right], \\ r &= \frac{f_{B_s}}{2f_\pi f_K} - \frac{f_{B_s}}{f_\pi f_K} \frac{p_B \cdot (p_2 - p_1)}{(p_B - p_1 - p_2)^2 - m_{B_s}^2} + \frac{2g f_{B^*}}{f_\pi f_K} \sqrt{\frac{m_{B_s}}{m_{B^*}}} \frac{(p_B - p_1) \cdot p_1}{s_{23} - m_{B^*}^2} \\ &\quad - \frac{4g^2 f_{B_s}}{f_\pi f_K} \frac{m_{B_s} m_{B^*}}{(p_B - p_1 - p_2)^2 - m_{B_s}^2} \frac{p_1 \cdot p_2 - p_1 \cdot (p_B - p_1) p_2 \cdot (p_B - p_1) / m_{B^*}^2}{s_{23} - m_{B^*}^2}, \end{aligned} \quad (9)$$

where $s_{ij} \equiv (p_i + p_j)^2$. g is a heavy-flavor independent strong coupling which has been extracted from the CLEO measurement of the D^{*+} decay width [32], $|g| = 0.59 \pm 0.01 \pm 0.07$. In this work, we also follow [23] and adopt its sign as negative. Thus, we drive the current-induced amplitude as:

$$\begin{aligned} A_{\text{current-ind}} &\equiv \langle \pi^-(p_3) | (\bar{d}u)_{V-A} | 0 \rangle \langle K^0(p_1) \pi^+(p_2) | (\bar{u}b)_{V-A} | B^- \rangle \\ &= -\frac{f_\pi}{2} [2m_3^2 r + (m_B^2 - s_{12} - m_3^2) \omega_+ + (s_{23} - s_{13} - m_2^2 + m_1^2) \omega_-] e^{-\alpha_{NR} p_B \cdot (p_1 + p_2)} e^{i\phi_{12}}. \end{aligned} \quad (10)$$

As stated in Sec. 1, the exponential form $e^{-\alpha_{NR} p_B \cdot (p_1 + p_2)}$ is introduced so that the HMChPT results are recovered in the soft meson region and

$$\alpha_{NR} = 0.081_{-0.009}^{+0.015} \text{ GeV}^{-2}, \quad (11)$$

which is constrained from the tree dominated decay $B^- \rightarrow \pi^+ \pi^- \pi^-$.² The unknown strong phase ϕ_{12} is set to be zero for simplicity.

In this decay mode, vector meson (K^*) and scalar resonances ($K_0^*(1430)$) also contribute to the three-body matrix element $\langle K^0(p_1) \pi^+(p_2) | (\bar{u}b)_{V-A} | \overline{B}_s^0(p_B) \rangle$, whose effects are described in terms of the usual Breit-Wigner

¹In the chiral limit, $\langle K^0 \pi^+ \pi^- | (\bar{s}d)_{V-A} | 0 \rangle$ has been proven to be zero [9]. For the term $\langle K^0 \pi^+ \pi^- | \bar{s}(1 + \gamma_5)d | 0 \rangle$, it is penguin induced and power suppressed. Thus, its contribution could be dropped safely.

²In the above calculations, the heavy-quark chiral effective approach has been adopted, where the light pseudoscalar mesons are regarded as Goldstone bosons. Thus, the $SU(3)$ symmetry breaking effects in α_{NR} have not been involved for their negligible uncertainties.

formalism. So, we have the expression as

$$\begin{aligned} \langle K^0(p_1)\pi^+(p_2)|(\bar{u}b)_{V-A}|\bar{B}_s^0\rangle^R &= \frac{g^{K^{*+}\rightarrow K^0\pi^+}}{s_{12}-m_{K^{*+}}^2+im_{K^{*+}}\Gamma_{K^{*+}}}\sum_{\text{pol}}\varepsilon^*\cdot(p_1-p_2)\langle K^{*+}|(\bar{u}b)_{V-A}|\bar{B}_s^0\rangle \\ &- \frac{g^{K_0^{*+}\rightarrow K^0\pi^+}}{s_{12}-m_{K_0^{*+}}^2+im_{K_0^{*+}}\Gamma_{K_0^{*+}}}\langle K_0^{*+}|(\bar{u}b)_{V-A}|\bar{B}_s^0\rangle, \end{aligned} \quad (12)$$

where we have ignored the contribution of $K^*(1410), K^*(1680), \dots$. Hence,

$$\begin{aligned} &\langle K^0(p_1)\pi^+(p_2)|(\bar{u}b)_{V-A}|\bar{B}_s^0\rangle^R \langle \pi^-(p_3)|(\bar{d}u)_{V-A}|0\rangle \\ &= -f_\pi \frac{g^{K^{*+}\rightarrow K^0\pi^+}}{s_{12}-m_{K^*}^2+im_{K^*}\Gamma_{K^*}} \left\{ \left[s_{13}-s_{23} + \frac{(m_{B_s}^2-m_\pi^2)(m_\pi^2-m_K^2)}{m_{K^*}^2} \right] [m_{K^*}A_0^{B_s K^*}(q^2) \right. \\ &\quad + \frac{A_2^{B_s K^*}(q^2)}{2(m_{B_s}+m_{K^*})}(s_{12}-m_{K^*}^2)] + (m_\pi^2-m_K^2) \left(1 - \frac{s_{12}}{m_{K^*}^2} \right) [m_{K^*}A_0^{B_s K^*}(q^2) \\ &\quad \left. - (m_{B_s}+m_{K^*})A_0^{B_s K^*}(q^2) + \frac{A_2^{B_s K^*}(q^2)}{2(m_{B_s}+m_{K^*})}(s_{12}-m_{K^*}^2) \right] \Big\} \\ &\quad + f_K \frac{g^{K_0^{*+}\rightarrow K^-\pi^+}}{s_{12}-m_{K_0^*}^2+im_{K_0^*}\Gamma_{K_0^*}} \left[(m_{B_s}^2-m_{K_0^*}^2)F_0^{B_s K_0^*}(q^2) + (m_{K_0^*}^2-s_{12})F_1^{B_s K_0^*}(q^2) \right]. \end{aligned} \quad (13)$$

with $q^2 = (p_B - p_1 - p_2)^2 = p_3^2$. In the above formulae, the definitions of decay constants and form factors are referred to Refs. [13, 33, 34].

For the transition processes that are penguin induced or color suppressed, because the time-like e.m. form factors of two pions have not been measured well enough, we will thus ignore the nonresonant contributions and only consider the contributions from the vector and scalar mesons. Hence, the amplitude of the transition process is read as

$$\begin{aligned} &\langle \pi^+(p_2)\pi^-(p_3)|(\bar{u}u)_{V-A}|0\rangle^R \langle K^0(p_1)|(\bar{d}b)_{V-A}|\bar{B}_s^0\rangle = -F_1^{B_s K}(s_{23})F_R^{\pi^+\pi^-}(s_{23})(s_{12}-s_{13}), \\ &\langle \pi^+(p_2)\pi^-(p_3)|\bar{d}d|0\rangle^R \langle K^0(p_1)|\bar{d}b|\bar{B}_s^0\rangle = -\frac{m_{B_s}^2-m_K^2}{m_b-m_d}F_0^{B_s K}(s_{23})\sum_i \frac{m_{f_{0i}}\bar{f}_{f_{0i}}^d g^{f_{0i}\rightarrow\pi^+\pi^-}}{s_{23}-m_{f_{0i}}^2+im_{f_{0i}}\Gamma_{f_{0i}}}, \end{aligned} \quad (14)$$

with the definition of the form factor $F_R^{\pi^+\pi^-}$:

$$F_R^{\pi^+\pi^-}(s) = \frac{1}{\sqrt{2}}\sum_i \frac{m_{\rho_i}f_{\rho_i}g^{\rho_i\rightarrow\pi^+\pi^-}}{s-m_{\rho_i}^2+im_{\rho_i}\Gamma_{\rho_i}}, \quad (15)$$

where $\rho_i = \rho, \rho(1450), \dots$ and $f_0 = f_0(980), f_0(1370), f_0(1500), \dots$. The scalar decay constant $\bar{f}_{f_{0i}}^q$ is defined by $\langle f_{0i}|\bar{q}q|0\rangle = m_{f_{0i}}\bar{f}_{f_{0i}}^q$, and $g^{f_{0i}\rightarrow\pi^+\pi^-}$ is the strong coupling of the $f_{0i} \rightarrow \pi^+\pi^-$ decay. In the practical numerical calculations, the higher excited states of vector mesons have been ignored for their negligible contributions.

For the scalar meson $f_0(980)$, we will consider it as the conventional $q\bar{q}$, though the quark structure of the light scalar mesons below or near 1 GeV has been quite controversial. Because some experimental evidences indicate that $f_0(980)$ is not purely an $s\bar{s}$ state [35], we write the flavor wave functions of the $f_0(980)$ as:

$$|f_0(980)\rangle = |s\bar{s}\rangle \cos\theta + |n\bar{n}\rangle \sin\theta, \quad (16)$$

with $n\bar{n} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. Experimental implications for the mixing angle have been discussed in detail in [36]. By assuming 2-quark bound state for $f_0(980)$, the observed large rates of $B \rightarrow f_0(980)K$ and $f_0(980)K^*$ modes can be explained in QCDF with the mixing angle θ in the vicinity of 20° [37]. So, we use $\theta = 20^\circ$ in this work.

2.3 $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$

The factorizable $\bar{B}_s^0 \rightarrow K^0 K^+ K^-$ decay amplitude is given by

$$\begin{aligned}
\langle K^0 K^+ K^- | T_p | \bar{B}_s^0 \rangle &= \langle K^+ K^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0 | (\bar{d}s)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p - r_\chi^K (a_6^p - \frac{1}{2} a_8^p) \right] \\
&+ \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+ K^- | (\bar{u}u)_{V-A} | 0 \rangle \left[a_2 \delta_{pu} + a_3 + a_5 + a_7 + a_9 \right] \\
&+ \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+ K^- | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 + a_4^p + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}^p) \right] \\
&+ \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+ K^- | (\bar{s}s)_{V-A} | 0 \rangle \left[a_3 + a_5 - \frac{1}{2} (a_7 + a_9) \right] \\
&+ \langle K^+ | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^- K^0 | (\bar{d}u)_{V-A} | 0 \rangle \left[a_1 \delta_{pu} + a_4^p + a_{10}^p \right] \\
&+ \langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle K^+ K^- | \bar{d}d | 0 \rangle \left[-2a_6^p + a_8^p \right] \\
&+ \langle K^+ | \bar{u}b | \bar{B}_s^0 \rangle \langle K^- K^0 | \bar{d}u | 0 \rangle \left[-2a_6^p - 2a_8^p \right] \\
&+ \langle K^0 K^+ K^- | (\bar{d}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] \\
&+ \langle K^0 K^+ K^- | \bar{d}(1 + \gamma_5)s | 0 \rangle \langle 0 | \bar{s}\gamma_5 b | \bar{B}_s^0 \rangle \left[2a_6^p - a_8^p \right]. \tag{17}
\end{aligned}$$

For the current-induced process with a kaon emission, the form factors r and ω_\pm for the three-body matrix element $\langle K^+ K^- | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle$ evaluated in the framework of HMChPT are similar to that of Eq.(9) except that f_π is replaced by f_K . This process also receives the contributions of vector (ϕ) and scalar (f_0) resonants by

$$\begin{aligned}
&\langle K^+(p_2) K^-(p_3) | (\bar{s}b)_{V-A} | \bar{B}_s^0 \rangle^R \langle K^0(p_1) | (\bar{d}s)_{V-A} | 0 \rangle \\
&= -\frac{f_K}{2} \frac{g^{\phi \rightarrow K^+ K^-}}{s_{23} - m_\phi^2 + im_\phi \Gamma_\phi} (s_{12} - s_{13}) \left[(m_{B_s} + m_\phi) A_1^{B_s \phi}(q^2) - \frac{A_2^{B_s \phi}(q^2)}{m_{B_s} + m_\phi} (m_{B_s}^2 - s_{23}) \right. \\
&\quad \left. - 2m_\phi [A_3^{B_s \phi}(q^2) - A_0^{B_s \phi}(q^2)] \right] + f_K \sum_i \frac{g^{f_{0i} \rightarrow K^+ K^-}}{s_{23} - m_{f_{0i}}^2 + im_{f_{0i}} \Gamma_{f_{0i}}} F_0^{B_s f_{0i}}(q^2) (m_{B_s}^2 - s_{23}). \tag{18}
\end{aligned}$$

For the transition amplitude, in addition to the $b \rightarrow u$ tree transition, we need to consider the nonresonant contributions to the $b \rightarrow s$ penguin amplitude

$$A_1 = \langle K^0(p_1) | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^+(p_2) K^-(p_3) | (\bar{q}q)_{V-A} | 0 \rangle, \tag{19}$$

$$A_2 = \langle K^+(p_2) | (\bar{u}b)_{V-A} | \bar{B}_s^0 \rangle \langle K^0(p_1) K^-(p_3) | (\bar{d}u)_{V-A} | 0 \rangle, \tag{20}$$

$$A_3 = \langle K^0(p_1) | \bar{d}b | \bar{B}_s^0 \rangle \langle K^+(p_2) K^-(p_3) | \bar{d}d | 0 \rangle, \tag{21}$$

$$A_4 = \langle K^+(p_2) | \bar{u}b | \bar{B}_s^0 \rangle \langle K^0(p_1) K^-(p_3) | \bar{d}u | 0 \rangle. \tag{22}$$

We firstly calculate the two-kaon creation matrix element A_1 , which could be expressed in terms of the time-like kaon current form factors as

$$\langle K^+(p_2) K^-(p_3) | \bar{q}\gamma_\mu q | 0 \rangle = (p_{K^+} - p_{K^-})_\mu F_q^{K^+ K^-}. \tag{23}$$

The weak vector form factor $F_q^{K^+K^-}$ is related to the kaon electromagnetic (e.m.) form factors $F_{\text{em}}^{K^+K^-}$. Phenomenologically, the e.m. form factors receive resonant and nonresonant contributions and can be expressed by

$$F_{\text{em}}^{K^+K^-} = F_\rho^{KK} + F_\omega^{KK} + F_\phi^{KK} + F_{NR}. \quad (24)$$

It follows from Eqs. (23) and (24) that

$$\begin{aligned} F_u^{K^+K^-} &= F_\rho^{KK} + 3F_\omega^{KK} + \frac{1}{3}(3F_{NR} - F'_{NR}), \\ F_d^{K^+K^-} &= -F_\rho^{KK} + 3F_\omega^{KK}, \\ F_s^{K^+K^-} &= -3F_\phi^{KK} - \frac{1}{3}(3F_{NR} + 2F'_{NR}), \end{aligned} \quad (25)$$

where the isospin symmetry has been used. The resonant and nonresonant terms can be parameterized as $F_h(s_{23})$ and $F_{NR}^{(\prime)}(s_{23})$, respectively. Since their expressions have been given explicitly in Refs. [10, 11, 12, 13], we will not list them here. With the equation of motion, we therefore obtain:

$$A_1 = (s_{12} - s_{13})F_1^{B_sK}(s_{23})F_q^{K^+K^-}(s_{23}). \quad (26)$$

In A_3 , although the nonresonant contribution vanishes as both K^+ and K^- do not contain the valence d or \bar{d} quark, this matrix element does receive the contribution from the scalar f_0 pole,

$$\langle K^+(p_2)K^-(p_3)|\bar{d}d|0\rangle^R \equiv f_d^{K^+K^-}(s_{23}) = \sum_i \frac{m_{f_{0i}}\bar{f}_{f_{0i}}^d g^{f_{0i} \rightarrow K^+K^-}}{m_{f_{0i}}^2 - s_{23} - im_{f_{0i}}\Gamma_{f_{0i}}}, \quad (27)$$

which leads to

$$A_3 = \frac{m_B^2 - m_K^2}{m_b - m_s} F_0^{B_sK}(s_{23}) f_d^{K^+K^-}(s_{23}). \quad (28)$$

For the equations A_2 and A_4 , the contributions from nonresonant could be parameterized as F_{NR} and f_d^{NR} respectively by using $SU(3)$ symmetry. The formulae of f_d^{NR} is expressed and discussed in detail in [13]. After calculation, we obtain

$$A_2 = (s_{12} - s_{23})F_1^{B_sK}(s_{13})F_{NR}(s_{13}), \quad (29)$$

$$A_4 = \frac{m_B^2 - m_K^2}{m_b - m_s} F_0^{B_sK}(s_{13}) f_d^{NR}(s_{13}). \quad (30)$$

2.4 $\overline{B}_s^0 \rightarrow K^0 K^- \pi^+$ and $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$

The factorizable amplitudes of the $\overline{B}_s^0 \rightarrow K^0 K^- \pi^+$ and $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$ are given as :

$$\begin{aligned}
\langle K^0 K^- \pi^+ | T_p | \overline{B}_s^0 \rangle &= \langle K^0 \pi^+ | (\bar{u}b)_{V-A} | \overline{B}_s^0 \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - r_\chi^K (a_6^p + a_8^p)] \\
&+ \langle K^0 | (\bar{d}b)_{V-A} | \overline{B}_s^0 \rangle \langle K^- \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p \right] \\
&+ \langle K^0 | \bar{d}b | \overline{B}_s^0 \rangle \langle K^- \pi^+ | \bar{d}d | 0 \rangle \left[-2a_6^p + a_8^p \right] \\
&+ \langle K^0 K^- \pi^+ | (\bar{u}u)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle [a_2 \delta_{pu} + a_3 + a_9] \\
&+ \langle K^0 K^- \pi^+ | (\bar{d}d)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle \left[a_3 - \frac{1}{2} a_9 \right] \\
&+ \langle K^0 K^- \pi^+ | (\bar{s}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle \left[a_3 + a_4^p - \frac{1}{2} a_9 - \frac{1}{2} a_{10}^p \right] \\
&+ \langle K^0 \pi^+ \pi^- | \bar{s}(1 + \gamma_5) s | 0 \rangle \langle 0 | \bar{s} \gamma_5 b | \overline{B}_s^0 \rangle [2a_6^p - a_8^p], \tag{31}
\end{aligned}$$

$$\begin{aligned}
\langle \overline{K}^0 K^+ \pi^- | T_p | \overline{B}_s^0 \rangle &= \langle K^+ \pi^- | (\bar{d}b)_{V-A} | \overline{B}_s^0 \rangle \langle \overline{K}^0 | (\bar{s}d)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p - r_\chi^K (a_6^p - \frac{1}{2} a_8^p) \right] \\
&+ \langle K^+ | (\bar{u}b)_{V-A} | \overline{B}_s^0 \rangle \langle \overline{K}^0 \pi^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p] \\
&+ \langle K^+ | \bar{u}b | \overline{B}_s^0 \rangle \langle \overline{K}^0 \pi^- | \bar{s}u | 0 \rangle \left[-2a_6^p - 2a_8^p \right] \\
&+ \langle \overline{K}^0 K^+ \pi^- | (\bar{u}u)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle [a_2 \delta_{pu} + a_3 + a_9] \\
&+ \langle \overline{K}^0 K^+ \pi^- | (\bar{d}d)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle \left[a_3 - \frac{1}{2} a_9 \right] \\
&+ \langle \overline{K}^0 K^+ \pi^- | (\bar{s}s)_{V-A} | 0 \rangle \langle 0 | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle \left[a_3 + a_4^p - \frac{1}{2} a_9 - \frac{1}{2} a_{10}^p \right] \\
&+ \langle \overline{K}^0 K^+ \pi^- | \bar{s}(1 + \gamma_5) s | 0 \rangle \langle 0 | \bar{s} \gamma_5 b | \overline{B}_s^0 \rangle [2a_6^p - a_8^p], \tag{32}
\end{aligned}$$

For the current-induced processes, the three-body matrix elements $\langle K\pi | (\bar{q}b)_{V-A} | \overline{B}_s^0 \rangle$ have the similar expressions as Eqs.(9) and (10). Furthermore, these processes also receive resonant contributions, which is similar to Eq.(13) except that the symbols of the final mesons are exchanged. For the two-body matrix element $\langle K^- \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle$, we note that

$$\langle K^-(p_1) \pi^+(p_2) | (\bar{s}d)_{V-A} | 0 \rangle = (p_1 - p_2)_\mu F_1^{K\pi}(s_{12}) + \frac{m_K^2 - m_\pi^2}{s_{12}} (p_1 + p_2)_\mu \left[-F_1^{K\pi}(s_{12}) + F_0^{K\pi}(s_{12}) \right], \tag{33}$$

The resonant contributions are expressed by:

$$\begin{aligned}
\langle K^-(p_1) \pi^+(p_2) | (\bar{s}d)_{V-A} | 0 \rangle^R &= \sum_i \frac{g_{K_i^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_i^*}^2 + im_{K_i^*} \Gamma_{K_i^*}} \sum_{\text{pol}} \varepsilon^* \cdot (p_1 - p_2) \langle K_i^* | (\bar{s}d)_{V-A} | 0 \rangle \\
&- \sum_i \frac{g_{K_{0i}^* \rightarrow K^- \pi^+}}{s_{12} - m_{K_{0i}^*}^2 + im_{K_{0i}^*} \Gamma_{K_{0i}^*}} \langle K_{0i}^* | (\bar{s}d)_{V-A} | 0 \rangle. \tag{34}
\end{aligned}$$

Hence, form factors $F_1^{K\pi}$ and $(-F_1^{K\pi} + F_0^{K\pi})$ receive the following resonant contributions

$$\begin{aligned}
(F_1^{K\pi}(s))^R &= \sum_i \frac{m_{K_i^*} f_{K_i^*} g_{K_i^* \rightarrow K\pi}}{m_{K_i^*}^2 - s - im_{K_i^*} \Gamma_{K_i^*}}, \\
(-F_1^{K\pi}(s) + F_0^{K\pi}(s))^R &= \sum_i \frac{f_{K_{0i}^*} g_{K_{0i}^* \rightarrow K\pi}}{m_{K_{0i}^*}^2 - s - im_{K_{0i}^*} \Gamma_{K_{0i}^*}} \frac{s_{12}}{m_K^2 - m_\pi^2} - \sum_i \frac{m_{K_i^*} f_{K_i^*} g_{K_i^* \rightarrow K\pi}}{m_{K_i^*}^2 - s - im_{K_i^*} \Gamma_{K_i^*}} \frac{s_{12}}{m_{K_i^*}^2}. \tag{35}
\end{aligned}$$

As a result, the amplitude $\langle K^- \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle \langle K^0 | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle$ has the expression

$$\begin{aligned} & \langle K^-(p_1) \pi^+(p_2) | (\bar{s}d)_{V-A} | 0 \rangle \langle K^0(p_3) | (\bar{d}b)_{V-A} | \bar{B}_s^0 \rangle \\ &= F_1^{BsK}(s_{12}) F_1^{K\pi}(s_{12}) \left[s_{23} - s_{13} - \frac{(m_B^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{12}} \right] + F_0^{BsK}(s_{12}) F_0^{K\pi}(s_{12}) \frac{(m_B^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{12}}, \end{aligned} \quad (36)$$

where the momentum dependence of the weak form factor $F^{K\pi}(q^2)$ is parameterized as

$$F^{K\pi}(q^2) = \frac{F^{K\pi}(0)}{1 - q^2/\Lambda_\chi^2 + i\Gamma_R/\Lambda_\chi}, \quad (37)$$

with $\Gamma_R = 200$ MeV [9] being the width of the relevant resonance and $\Lambda_\chi = 0.83$ GeV being a chiral symmetry breaking scale.

For the term $\langle K\pi | \bar{s}d | 0 \rangle$, it receives contributions of both resonant and nonresonant, the expression of which is shown as

$$\langle K^-(p_1) \pi^+(p_2) | \bar{s}d | 0 \rangle = \frac{m_{K_0^*} \bar{f}_{K_0^*} g^{K_0^* \rightarrow K^- \pi^+}}{m_{K_0^*}^2 - s_{12} - im_{K_0^*} \Gamma_{K_0^*}} + \langle K^-(p_1) \pi^+(p_2) | \bar{s}d | 0 \rangle^{NR}. \quad (38)$$

In the above equation, the unknown two-body matrix elements of scalar densities $\langle K\pi | \bar{s}q | 0 \rangle$ are related to $\langle K^+ K^- | \bar{s}s | 0 \rangle$ via SU(3) symmetry, e.g.

$$\langle K^-(p_1) \pi^+(p_2) | \bar{s}d | 0 \rangle^{NR} = \langle K^+(p_1) K^-(p_2) | \bar{s}s | 0 \rangle^{NR} = f_s^{NR}(s_{12}), \quad (39)$$

with the expression of f_s^{NR} given as

$$f_s^{NR} = \langle K^-(p_1) \pi^+(p_2) | \bar{s}d | 0 \rangle^{NR} = \frac{m_K^2 - m_\pi^2}{m_s - m_d} (F_{NR} + \frac{2}{3} F'_{NR}) + \sigma_{NR} e^{-\alpha s_{12}}, \quad (40)$$

where $\sigma_{NR} = e^{i\pi/4} (3.36_{-0.96}^{+1.12})$ GeV is fixed from data of $\bar{B}^0 \rightarrow K_S K_S K_S$ [4]. If we adopt this value directly, we will get unexpected large branching fractions of $\bar{B}_s^0 \rightarrow K_S K^\mp \pi^\pm$, which means that final states interaction and SU(3) symmetry violation may be important. Thus, we phenomenologically introduce a factor $\beta = 0.8 \pm 0.1$, which stands for the effects of final states interaction and SU(3) symmetry violation. While in Ref.[13], a strong phase has been also introduced in order to describe this effect. As a result, we could obtain:

$$\langle K^0 | \bar{d}b | \bar{B}_s^0 \rangle \langle K^- \pi^+ | \bar{s}d | 0 \rangle = \frac{m_{B_s}^2 - m_K^2}{m_b - m_d} F_0^{BsK}(s_{12}) \left[\frac{m_{K_0^*} \bar{f}_{K_0^*} g^{K_0^* \rightarrow K^- \pi^+}}{m_{K_0^*}^2 - s_{12} - im_{K_0^*} \Gamma_{K_0^*}} + \beta f_s^{NR} \right]. \quad (41)$$

3 Numerical Results

To proceed with the numerical calculations, we firstly specify the parameters used in this work. For the CKM matrix elements, we use the updated Wolfenstein parameters $A = 0.823$, $\lambda = 0.22457$, $\bar{\rho} = 0.1289$ and $\bar{\eta} = 0.348$ [38]. The corresponding CKM angles are $\sin 2\beta = 0.689 \pm 0.019$ and $\gamma = (69.7_{-2.8}^{+1.3})^\circ$. The form factors used in this work are calculated within the covariant light-front quark model [39, 40], which are summarized as follows

$$\begin{aligned} & V^{B_s \rightarrow \phi}(0) = 0.23, A_0^{B_s \rightarrow \phi}(0) = 0.31, A_1^{B_s \rightarrow \phi}(0) = 0.25, A_2^{B_s \rightarrow \phi}(0) = 0.22, \\ & V^{B_s \rightarrow K^*}(0) = 0.23, A_0^{B_s \rightarrow K^*}(0) = 0.25, A_1^{B_s \rightarrow K^*}(0) = 0.19, A_2^{B_s \rightarrow K^*}(0) = 0.16, \\ & F_0^{B_s \rightarrow K}(0) = 0.24, F_0^{B_s \rightarrow K_0^*(1430)}(0) = 0.25, F_0^{B_s \rightarrow f_0^s}(0) = 0.28. \end{aligned} \quad (42)$$

The momentum dependence of form factors in the spacelike region can be well parameterized and reproduced in the following three-parameter form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_{B_s}^2) + b(q^2/m_{B_s}^2)^2} \quad (43)$$

where F stands for the relevant form factors and parameters a and b have been given explicitly in ref.[40].

In practical calculation, we shall assign the form factor error to be 0.03. For the strong coupling constants, most of them have been determined from the measured partial width in refs.[12, 13], which are shown as

$$\begin{aligned} g^{\rho(770) \rightarrow \pi^+ \pi^-} &= 6.0, \quad g^{K^*(892) \rightarrow K^+ \pi^-} = 4.59, \quad g^{f_0(980) \rightarrow \pi^+ \pi^-} = 1.18 \text{ GeV}, \quad g^{K_0^*(1430) \rightarrow K^+ \pi^-} = 3.84 \text{ GeV}, \\ g^{\phi \rightarrow K^+ K^-} &= -4.54, \quad g^{f_0(980) \rightarrow K^+ K^-} = 3.7 \text{ GeV}, \quad g^{f_0(1500) \rightarrow K^+ K^-} = 0.69 \text{ GeV}, \quad g^{f_0(1710) \rightarrow K^+ K^-} = 1.6 \text{ GeV}. \end{aligned} \quad (44)$$

For the running quark masses we shall use [35, 41]

$$\begin{aligned} m_b(m_b) &= 4.2 \text{ GeV}, & m_b(2.1 \text{ GeV}) &= 4.94 \text{ GeV}, & m_b(1 \text{ GeV}) &= 6.34 \text{ GeV}, \\ m_c(m_b) &= 0.91 \text{ GeV}, & m_c(2.1 \text{ GeV}) &= 1.06 \text{ GeV}, & m_c(1 \text{ GeV}) &= 1.32 \text{ GeV}, \\ m_s(2.1 \text{ GeV}) &= 95 \text{ MeV}, & m_s(1 \text{ GeV}) &= 118 \text{ MeV}, \\ m_d(2.1 \text{ GeV}) &= 5.0 \text{ MeV}, & m_u(2.1 \text{ GeV}) &= 2.2 \text{ MeV}. \end{aligned} \quad (45)$$

With above parameters and formulas in Sec.2, we calculated the branching fractions of resonant and nonresonant contributions to the decay modes concerned and presented them in Table.1. The theoretical errors are from the uncertainties in (i) the parameter α_{NR} which governs the momentum dependence of the nonresonant amplitude, (ii) the strange quark mass m_s , the form factors, the nonresonant parameter σ_{NR} and $SU(3)$ asymmetry violation parameter β , and (iii) the unitarity angle γ .

From Table. 1 we see that the decay $\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ is tree dominated and its main contribution arises from the K^{*+} meson, while the nonresonant contribution is less important. Compared with experimental data, the calculated branching fraction agrees well with the recent LHCb measurement. As for $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$, although it receives the color-suppressed tree contribution, it is dominated by transition $b \rightarrow d \bar{q} q$. Consequently, it has a small branching fraction $(2.29_{-0.01}^{+0.01} + 1.17_{-0.78}^{+0.05}) \times 10^{-6}$, which is much smaller than that of $\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$. Note that this decay is governed by the nonresonant background dominated by σ_{NR} . Hence this decay mode could be an ideal plat for constraining the unknown parameter σ_{NR} in turn. Experimentally, however, no significant evidence of this decay mode has been obtained, and its branching fraction is described in $(0.2 - 3.4) \times 10^{-6}$ at 90% confidence level (CL) based on the CL inferences in Ref. [42]. Obviously, the result we predicted is falling into the experimental range. We hope this decay will be measured precisely in the current LHCb experiment. The results of above two decay modes also confirm the conclusion that nonresonant decays play a prominent role in the penguin-dominated three-body B meson decays in Ref. [12].

For the decay $\overline{B}_s^0 \rightarrow K^0 K^- \pi^+$, the current-induced process with a K^- emission is tree dominated, while the transition processes $\langle \overline{B}_s^0 \rightarrow K^0 \rangle \times \langle 0 \rightarrow K^- \pi^+ \rangle$ are induced by penguin operators. On the contrary, the current-induced process of decay $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$ with a neutral kaon emission is induced by penguin, and the transition

Table 1: Branching fractions (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $\overline{B}_s^0 \rightarrow K_s^0 h^+ h'^-$.

Decay mode	Theory	Decay mode	Theory
$\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$			
$K^{*+} \pi^-$	$7.72^{+0.00+1.44+0.04}_{-0.00-1.27-0.04}$	$K^0 f_0(980)$	$0.25^{+0.00+0.09+0.01}_{-0.00-0.07-0.00}$
$K_0^{*+}(1430) \pi^-$	$2.91^{+0.00+0.77+0.02}_{-0.00-0.67-0.02}$	$K^0 f_0(1370)$	$0.25^{+0.00+0.07+0.00}_{-0.00-0.06-0.01}$
$K^0 \rho^0$	$0.53^{+0.00+0.14+0.01}_{-0.00-0.13-0.01}$	NR	$2.90^{+0.68+0.37+0.05}_{-0.77-0.29-0.05}$
Total	$12.58^{+0.49+2.42+0.10}_{-0.65-2.08-0.11}$		
$\overline{B}_s^0 \rightarrow K^0 K^+ K^-$			
ϕK^0	$0.18^{+0.00+0.15+0.01}_{-0.00-0.08-0.01}$	$f_0(980) K^0$	$0.20^{+0.00+0.16+0.00}_{-0.00-0.08-0.00}$
$f_0(1500) K^0$	$0.10^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$	NR	$1.87^{+0.01+0.71+0.04}_{-0.01-0.59-0.05}$
Total	$2.29^{+0.01+1.17+0.05}_{-0.01-0.78-0.05}$		
$\overline{B}_s^0 \rightarrow K^0 K^- \pi^+$			
$K^{*+} K^-$	$1.27^{+0.00+2.03+0.03}_{-0.00-0.73-0.04}$	$\overline{K}^{*0} K^0$	$2.27^{+0.00+0.60+0.01}_{-0.00-0.53-0.01}$
$K_0^{*+}(1430) K^-$	$0.89^{+0.00+1.43+0.02}_{-0.00-0.51-0.02}$	$\overline{K}_0^{*0}(1430) K^0$	$16.63^{+0.00+5.12+0.02}_{-0.00-4.32-0.03}$
NR	$12.89^{+0.28+13.17+0.05}_{-0.36-6.56-0.05}$		
Total	$34.24^{+0.23+21.11+0.09}_{-0.35-11.93-0.08}$		
$\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$			
$K^{*0} \overline{K}^0$	$0.73^{+0.00+1.70+0.00}_{-0.00-0.53-0.00}$	$K^{*-} K^+$	$2.27^{+0.00+0.60+0.00}_{-0.00-0.54-0.00}$
$K_0^{*0}(1430) \overline{K}^0$	$0.51^{+0.00+1.20+0.00}_{-0.00-0.37-0.00}$	$K_0^{*-}(1430) K^+$	$15.47^{+0.00+4.55+0.00}_{-0.00-3.89-0.00}$
NR	$12.29^{+0.25+12.58+0.02}_{-0.32-6.29-0.02}$		
Total	$33.71^{+0.15+20.93+0.01}_{-0.19-11.95-0.01}$		

processes receive the effects not only from tree but from penguin operators. In these two decays, the nonresonant contributions arise dominantly from the transition process via the scalar density $\langle K \pi | \bar{s} q | 0 \rangle$, and slightly from the current-induced process. Thus, the nonresonant contributions are sensitive to the matrix elements of scalar densities f_s^{NR} , as shown in Table.1. For the resonant contributions, both of them are dominated by the scalar particles $K_0^*(1430)$. Considering the parameter β standing for effects of the $SU(3)$ symmetry violation and the final states rescattering, the sum of two branching fractions is $(67.95^{+0.38+42.04+0.09}_{-0.54-23.88-0.08}) \times 10^{-6}$, which could accommodate data of the recent LHC measurement well. We hope these two decays could be measured individually in the future experiment.

In QCD calculations based on a heavy quark expansion, one faces uncertainties arising from power corrections

Table 2: Direct CP asymmetries (in %) for decay modes of \overline{B}_s^0 decays.

Final state	Total	Nonresonant
$\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$	$10.4^{+0.4+0.4+0.1}_{-0.5-0.7-0.1}$	$23.6^{+1.6+0.8+0.2}_{-1.6-1.6-0.2}$
$\overline{B}_s^0 \rightarrow K^0 K^+ K^-$	$-16.6^{+0.1+0.6+0.1}_{-0.2-0.5-0.1}$	$-19.4^{+0.0+0.1+0.2}_{-0.0-0.1-0.2}$
$\overline{B}_s^0 \rightarrow K^0 K^- \pi^+$	$-1.8^{+0.5+0.5+0.0}_{-0.5-0.6-0.1}$	$-2.2^{+0.5+1.1+0.1}_{-0.4-0.9-0.1}$
$\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$	$0.1^{+0.0+0.1+0.0}_{-0.0-0.3-0.0}$	$0.7^{+0.0+0.0+0.0}_{-0.0-0.0-0.0}$

such as annihilation and hard-scattering contributions. For example, in QCD factorization [31], there are large theoretical uncertainties related to the modelling of power corrections corresponding to weak annihilation effects and the chirally enhanced power corrections to hard spectator scattering. Even for two-body B decays, power corrections are of order $(10 - 20)\%$ for tree-dominated modes, but they are usually bigger than the central values for penguin-dominated decays. Needless to say, $1/m_b$ power corrections for three-body decays may well be larger. However, in the current work we use the phenomenological factorization model rather than in the established theories based on a heavy quark expansion. Consequently, uncertainties due to power corrections, at this stage, are not included in our calculations, by assumption. In view of such shortcomings we must emphasize that the additional errors due to such model dependent assumptions may be sizable.

In this work, the CP asymmetries of these four decays are also calculated, and the results are summarized in Table.2. We see from the table that the decay $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$ has large CP asymmetries with and without resonant contributions. Note that the two asymmetries have the same sign, as this decay is dominated by the nonresonant background, which can also be read from Table.1. For other three decays, the sizable resonant contributions may affect the CP asymmetries by taking large strong phases. In fact, the strong phases could arise from the effective Wilson coefficients, the Breit-Wigner formalism for resonances and the penguin matrix elements of scalar densities. Besides, the final states interactions may take new phases, which cannot be calculated directly up to now. Although the CP asymmetries of $B \rightarrow KKK, KK\pi$ [43, 44] have been measured in LHCb recently, the CP asymmetries of three-body of B_s^0 have not been explored till now. The CP asymmetries of these four decays are hoped to be measured in the current LHCb experiment or Super-b in the future, and they might be helpful to test the factorization approach in B_s^0 meson three-body decays.

4 Summary

Recently, LHCb collaboration published their first measurements of charmless three-body decays of B_s^0 meson, corresponding to an integrated luminosity of 1.0 fb^{-1} recorded at a centre-of-mass energy of 7 TeV. Motivated by this, we calculated the branching fractions of $\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$, $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$, $\overline{B}_s^0 \rightarrow K^0 \pi^+ K^-$ and $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$ decay modes within the factorization approach, which is generalized by Cheng *et al.* Both nonresonant contributions and resonant contributions have been studied in detail. For the decays $\overline{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ and $\overline{B}_s^0 \rightarrow$

$K^0 K^+ K^-$, our results agree well with experimental data. Especially, the former mode is dominated by the K^* and $K_0^*(1430)$ poles, while the latter is dominated by the nonresonant contribution. By adding the effects of the flavor $SU(3)$ symmetry violation, the sum of branching fractions of $\overline{B}_s^0 \rightarrow K^0 \pi^+ K^-$ and $\overline{B}_s^0 \rightarrow \overline{K}^0 K^+ \pi^-$ could accommodate the data. It should be emphasized that the branching fractions are very sensitive to the scalar density $\langle K\pi|\bar{s}q|0\rangle$. We hope these branching fractions could be measured individually in the experiments so as to test the factorization approach in three-body decays of \overline{B}_s^0 mesons. Moreover, the direct CP asymmetries of these decays have been also explored, and the sizable results could be measured in the running LHCb experiment and Super-b factory in the future.

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Note added

When this paper is being prepared, Hai-Yang Cheng and Chun-Khiang Chua posted their paper to the e-print archiv [45]. The same decays have been studied in that work, and most of our results agree with theirs after considering the differences of parameters (form factors). In [45], much attention is paid to the U -spin asymmetry, while in this work we paid much attention to disentangle the resonant and nonresonant contributions. Moreover, in dealing with the flavor $SU(3)$ symmetry violation of $\langle K\pi|0\rangle$, different approaches are adopted.

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